ECON 415 – Game Theory
Exercise 1: Strategic games with complete information

Ayça Özdoğan

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1. Answer the questions below for the following strategic game.

Table 1: Payoff Matrix

	L	M	R
U	$1,2$	$2,1$	$1,0$
C	$2,1$	$0,1$	$0,0$
D	$0,1$	$0,0$	$1,2$

   (a) What are the strictly dominated action for player 1 and player 2 (if there is any)?
   (b) What are the weakly dominated action for player 1 and player 2 (if there is any)?
   (c) What are the action profiles that survive Iterated Elimination of Strictly Dominated (IESD) actions?
   (d) What are the action profiles that survive Iterated Elimination of Weakly Dominated (IEWD) actions?
   (e) Find the pure Nash Equilibria.
   (f) Do all Nash equilibria survive IESD/IEWD actions?

2. Consider the following price competition. Two firms set prices in a market whose demand curve is given by

$$Q = 4 - P$$

where P is the lower of the two prices. If firm 1 is the lower priced firm, then it is firm 1 that meets all of the demand; conversely, the same applies to firm 2 if it is the lower priced firm. In case they post the same price they each get half of the market. The prices can be only quoted in terms of $TL units such as 0,1,2,3,4 TL. Suppose that the cost of production is zero for both firms and each firm wants to maximize its profits.

(a) Write down the strategic form and payoff matrix of this game.

(b) Is there a strictly dominant strategy equilibrium of this game? Explain.

(c) Is there a weakly dominant equilibrium of this game? Explain.

(d) What are the action profiles that survive Iterated Elimination of Strictly Dominated (IESD) actions? Explain.

(e) What are the action profiles that survive Iterated Elimination of Weakly Dominated (IEWD) actions? Explain.

(f) Is the game dominance solvable?

(g) Find the set of pure Nash Equilibria.

(h) Do Nash equilibria survive IESD/IEWD actions?

3. Consider the following auction scenario. Two individuals, player 1 and player 2, are competing to obtain a valuable object. Each player bids in a sealed envelope without knowing the bid of the other player. The bids must be in multiples of \$100 and the maximum amount to bid is \$500. The object is worth \$400 to player 1 and \$300 to player 2. The highest bidder wins the object. In case of a tie, player 1 gets the object. The winner of the object pays whatever she bids. If she doesn't win the object her payoff is zero.

(a) Write down the strategic form and payoff matrix of this game.

(b) Is there a strictly dominant strategy equilibrium of this game? Explain.

(c) Is there a weakly dominant equilibrium of this game? Explain.

(d) What are the action profiles that survive **Iterated Elimination of Strictly Dominated (IESD) actions**?

(e) What are the action profiles that survive Iterated Elimination of Weakly Dominated (IEWD) actions? Explain.

(f) Is the game dominance solvable?

(g) Find the Nash Equilibria.

(h) Do Nash equilibria survive IESD/IEWD actions?

4. There are two partners in a firm. Each partner chooses independently and simultaneously how much effort to put on the job. The total profit of the firm which the partners *share equally* is given by

$$\pi(x, y) = 4(x + y + \frac{1}{2}xy)$$

where x is the amount of effort chosen by partner 1 and y is the amount of effort chosen by partner 2. Assume that x and y have to be between 0 and 4. Partner 1's cost of effort is x^2 and partner 2's cost of effort is y^2. Each partner's payoff is given by his share of profits minus the cost of effort.

(a) Find and draw the best response correspondence for each partner. The payoffs to each player is half of the profit minus the cost:

$$u_1(x,y) = 2(x+y+\frac{1}{2}xy) - x^2$$
$$u_2(x,y) = 2(x+y+\frac{1}{2}xy) - y^2$$

(b) What are the Nash equilibrium levels of effort choices?

5. Tragedy of commons (public good problem): Suppose that there are two firms each choosing how much to produce *simultaneously*. Each production consumes some of the clean air. There is a total amount of clean air that is equal to K and the consumption of clean air comes out of this common resource. Each player i (firm) chooses its own consumption of clean air for production, which is denoted by $k_i \geq 0$. The amount of clean air left is $K - \sum_{j=1}^{2} k_j$. The firm enjoys not only the consumption of the clean air for its production but also the clean air left after the production. Thus, its payoff function is given as:

$$u_i(k_i, k_{-i}) = \ln(k_i) + \ln(K - \sum_{j=1}^{2} k_j)$$

Answer the questions below for this environment.

(a) Describe this situation as a strategic game.

(b) Compute and draw the best response correspondence for each firm. Then find the NE.

(c) Is the Nash equilibrium outcome Pareto efficient? If not, give an example of an efficient strategy profile.

(d) Which actions survive one round of iterated elimination of strictly dominated actions? What is the rationality requirement for one round of iteration? Justify your answer.

(e) Which strategy profiles survive IESDS? Is this game dominance solvable? What is the rationality requirement (rationality, k-level knowledge, common knowledge)? Justify your answer.

6. Consider **Cournot** duopoly market game with linear demand $P(Q) = 130 - Q$, where Q is the total quantity, i.e. $Q = q_1 + q_2$. Each firm has constant marginal cost c_i, $C_i(q_i) = 10q_i$ i.e. marginal costs are 10. Each firm *simultaneously* chooses a quantity level to produce.

(a) Write down the strategic form of this game.

(b) Derive and draw the best responses of each firm. Clearly label your graph. Find the Cournot-Nash equilibrium (q^c, q^c) of this game.

(c) Which choices survive one round of IESDS? What is the rationality requirement for one round of elimination? Justify your answer.

(d) Which choices survive IESDS? Is this game dominance solvable? What is the rationality requirement (rationality, k-level knowledge, common knowledge)? Justify your answer.

(e) Is the Cournot-Nash equilibrium Pareto efficient? Justify your answer.

7. **Cournot competition with n firms:** Consider the n-player Cournot oligopoly model (each firm i chooses the quantity it produces q_i simultaneously) with linear demand and cost functions:

$$p(Q) = \begin{cases} a - Q, & Q \leq a \\ 0 & Q > a, \end{cases}$$

where Q is the total output produced and for each i, $c_i(q_i) = cq_i$, where $a > c > 0$.

(a) Show that the unique and symmetric Nash equilibrium is

$$q_i = \frac{a-c}{n+1}, \quad \text{for every} \quad i = 1, ..., n.$$

(b) Show that $p \to c$ (the competitive-equilibrium price) as $n \to \infty$.

8. Bertrand competition with homogenous products: Suppose that there are two firms with unit costs $c > 0$. They choose prices for the same product they produce simultaneously. The one with the lower price captures the entire market. In case of a tie, they share the market equally. The total market demand is equal to 1.

(a) Write down the strategic form of this game.

(b) Compute and draw the best response correspondences. Find Nash equilibria.

9. Show or compute the followings for the first highest price auction. (You can assume there are only two players.)

(a) Truthtelling, i.e. $b_i = v_i$ is not a Nash equilibrium.

(b) Player 1 wins in all the Nash equilibria.

(c) $b_1^* > b_2^*$ cannot happen in a Nash equilibrium.(Thus, $b_1^* = b_2^*$ in every equilibrium.)

(d) Neither $b_1* < v_2$ nor $b_1* > v_1$ can hold. (Thus, in any Nash equilibrium of this game $v_2 \leq b_1^* = b_2^* \leq v_1$.)

(e) Find the set of Nash equilibria by drawing the best responses.

(f) Show that bidding anything strictly higher than v_2 is weakly dominated for player 2. So what is the undominated Nash equilibrium?