ECON 415 – Game Theory
Homework 1: Strategic (Normal) Form Games

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Homework assignment is out of 100 points. Randomly selected four questions will be graded. You have to submit it in until February 3, Friday, at midnight to *uzak* platform.

1. Consider the following two player game:

	L	M	R
T	$2, -1$	$4, 2$	$2, 0$
C	$3, 3$	$0, 0$	$1, 1$
B	$1, 2$	$2, 8$	$5, 1$

   (a) What are the strategies that survive IESDS?

   (b) At each step of the elimination what were the rationality assumptions?

   (c) Find all Nash equilibria including the mixed one.

2. Discrete First-Price Auction: An item is up for an auction. There are two players. Player 1 values the item at 3 while player 2 values the item at 5. Each player can bid either 0, 1 or 2. If player i bids more that player j then player i wins the item and pays his bid while the loser does not pay. If both players bid the same amount, then a coin is tossed to determine the winner, and the winner gets the item and pays his bid.

   (a) Write down this game as a normal-form game and in matrix form.

   (b) Does any player have strictly dominated strategy?

   (c) Which strategies survive IESDS? What is the rationality requirement?

   (d) Find the set of pure Nash equilibria. Does it survive IESDS?

3. **Domination with mixed strategies** Consider the following two-person game.

	L	R
T	$4,2$	$0,0$
M	$0,0$	$4,2$
B	$1,1$	$1,1$

(a) Let p be the probability player 1 plays T and q be the probability that player 2 plays L. What is the range of values p can take so that the mixture of T and M strictly dominates B, i.e. find the set of mixed strategies that strictly dominates B?

(b) Given that B is strictly dominated by a mixture of T and M, find and *draw* the best responses of each player and the set of all (pure and mixed) NE.

4. Tragedy of commons (public good problem): Suppose that there are two firms each choosing how much to produce *simultaneously*. Each production consumes some of the clean air. There is a total amount of clean air that is equal to K and the consumption of clean air comes out of this common resource. Each player i (firm) chooses its own consumption of clean air for production, which is denoted by $k_i \geq 0$. The amount of clean air left is $K - \sum_{j=1}^{2} k_j$. The firm enjoys not only the consumption of the clean air for its production but also the clean air left after the production. Thus, its payoff function is given as:

$$u_i(k_i, k_{-i}) = \ln(k_i) + \ln(K - \sum_{j=1}^{2} k_j)$$

Answer the questions below for this environment.

(a) Describe this situation as a strategic game.

(b) Compute and draw the best response correspondence for each firm. Then find the NE.

(c) Is the Nash equilibrium outcome Pareto efficient? If not, give an example of an efficient strategy profile.

(d) Which actions survive one round of iterated elimination of strictly dominated actions? What is the rationality requirement for one round of iteration? Justify your answer.

(e) Which strategy profiles survive IESDS? Is this game dominance solvable? What is the rationality requirement (rationality, k-level knowledge, common knowledge)? Justify your answer.

5. **Bertrand competition with homogenous products:** Suppose that there are two firms with unit costs $c > 0$. They choose prices for the same product they produce simultaneously. The one with the lower price captures the entire market. In case of a tie, they share the market equally. The total market demand is equal to 1.

   (a) Write down the strategic form of this game.

   (b) Compute and draw the best response correspondences. Find Nash equilibria.

6. State whether the following statements are true or false. Prove if it is true and give a counterexample if it is false.

   (a) A strictly dominated action profile cannot be Nash equilibrium.

   (b) A Nash equilibrium profile cannot involve a play of weakly dominated action.

   (c) Every finite normal form game has a pure strategy Nash equilibrium.

   (d) Every finite normal form game has a completely mixed (not pure) strategy Nash equilibrium.

7. **(War of Attrition)** Consider a situation where there are two parties disputing over an object. Assume that the value party i attaches to the object is $v_i > 0$. Let time be a continuous variable that starts from 0 and runs indefinitely. Each unit of time that passes before the dispute is settled (i.e. one of the parties concedes) costs each party one unit of payoff. Thus, if player i concedes first, at time t_i, her payoff is $-t_i$ (she spends t_i units of time and doesn't obtain the object.) If the other player concedes first, at time t_j, player i's payoff is $v_i - t_j$ (she obtains the object after t_j units of time). If both players concede at the same time, player i's payoff is $\frac{1}{2}v_i - t_i$ where t_i is the common concession time.

   (a) Write down the strategic form of this game.

   (b) Derive and draw the best response correspondences and find the set of Nash equilibria. (You can assume $v_1 > v_2$)