ECON 415 – Game Theory
Homework 2: Bayesian Games

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Homework assignment is out of 100+10 (bonus) points. All **four** questions and the **bonus** will be graded. You have to upload it to uzak.etu.edu.tr as one pdf file until March 10, Friday, at midnight.

1. Consider the following game:

	L	R
A	3,θ	0, 0
B	2,2 θ	2, θ
C	0,0	3, -θ

   where $\theta \in \{-1, 1\}$ is only known by player 2. Player 1 believes that $\theta = -1$ with probability $\frac{1}{2}$ and $\theta = 1$ with probability $\frac{1}{2}$.

   (a) Write this formally as a Bayesian game.

   (b) Find the Bayesian Nash equilibrium of this game.

2. Consider the strategic form game between a robber and a victim. Robber chooses to attack (A) or pass (P) and victim chooses to fight back (F) or yield (Y). Victim is weak or strong and the payoff matrices (where the robber is the row player and victim is column) is as follows (left for the weak and right for the strong victim). The robber does not know the type of the victim but believes that she is strong with probability $\mu \in (0, 1)$.

   (a) Find the pure strategy pooling equilibria.

   (b) Find the set of separating equilibria.

	F	Y
A	-1,-3	2, -2
P	0,1	0, 0

	F	Y
A	-1,-1	2, -2
P	0,1	0, 0

3. **The product-choice game:** Player 1 is a firm who can exert either high effort (H) or low effort (L) in the production of its output. Player 2 is a consumer who can buy either a high-priced product (h) or a low-priced product (l). Player 2 prefers high-priced product if the firm exerted high effort, but prefers the low-priced if the firm has not. The payoffs of this game is given by the table below:

   Table 1: Payoff Matrix

	h	l
H	2, 3	0,2
L	3, 0	1, 1

   Suppose that the consumer believes the firm could be one of two types: a *strategic* type who has the payoffs given in above matrix (with probability μ) and a *virtuous* type who finds it strictly optimal to exert high effort and thus committed to play H (with probability $1-\mu$). What is the Bayesian Nash equilirium of this game?

4. Consider the interaction between a regulator (he) and bank (she) where the possible actions for the bank are to be truthful or untruthful about its financial statements; and those for the regulator are to be lazy or diligent in auditing the bank. The expected payoffs of the players are given by table below. Row player is the bank who chooses to be truthful (T) or untruthful (U) and the column player is the regulator who chooses to be diligent (D) or lazy (L).

   Table 2: Payoff Matrix

	L	D
T	2, 3	0,2
U	3, 0	-1, 1

   Suppose that the regulator believes that the bank is a *honest* type with probability θ, who is committed to play truthfully (i.e. she finds it strictly dominant to play T) and strategic type with probability $1 - \theta$. Find the Bayesian Nash equilibrium of this game. Is there a pooling equilibrium?

5. BONUS Two bidders are involved in a first-price sealed bid auction where the value of the object is common $v > 0$ for both of them. Bidders simultaneously submit a bid, which can be any nonnegative number, and the highest bidder wins and pays her bid. In case of a tie, each bidder gets the object with an equal probability. If bidder i bids b_i and wins the object, then her payoff is $v - b_i$, while is she loses her payoff is 0.

   (a) Assume v is common knowledge. Write down the payoffs and drive the best response correspondence for each player. Find the Nash equilibria by finding the intersection of best responses.

   (b) Assume now v is not common knowledge. Player i observes only a signal t_i about v and the common value is equal to the sum of the signals $v = t_1 + t_2$. Each player knows only her own signal but believes that the other signal is distributed independently and has a uniform distribution over $[0, 1]$. Find a symmetric Bayesian equilibrium of this game in which player i's strategy is to bid $b_i(t_i) = at_i$ for some $a > 0$.(Hint: Show that there is no unilateral deviation for any type of each player.)