TOBB ETU Department of Economics
ECON 415 Game Theory
Midterm Exam

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Name:..

- The exam is out of 100 + 10 points.
- The exam ends at 15:20.
- You are not supposed to use a calculator, check your phones, look at your notes, the textbook or others' tests during the exam. Engaging in these activities are all considered as cheating in the exam.
- Please show all your work to get partial. credit.
- Allocate your time efficiently.
- Don't forget fully label all graphs.
- Make sure that your exam is 8 pages.

Good luck ⌣̈

1 Strategic games with complete information - 40 points

1.1 Definitions/True-False - 24 points

Consider a normal form game $G =< N, (S_i)_{i\in N}, (u_i)_{i\in N} >$ where $N = \{1, ..., n\}$ is the set of players, S_i is the strategy set for player $i \in N$ and $u_i : S \rightarrow \Re$ is the payoff function of player $i \in N$ (attaching a payoff to each strategy profile $s = (s_1, ..., s_n) \in S = \prod_{i\in N} S_i$). Let $s_{-i} \in \prod_{j\neq i} S_j$ denote a strategy profile of all players but player i. State whether the following statements are true or false. Prove if it is true and give a counter example if it is false. Define the terms that you may be using in proofs to get partial credit.

1. Every finite normal form game has a pure strategy Nash equilibrium.
   Her sonlu normal form oyunun pure Nash dengesi vardır.

2. Every finite normal form game has a completely mixed strategy Nash equilibrium.
   Her sonlu normal form oyunda tamamen mixed (pure olmayan) bir Nash dengesi vardır.

3. A strictly dominant action cannot be a completely mixed action.
   Herhangi bir strictly dominant (kesin basan) action profili tamamen mixed (saf olmayan) bir hareket olamaz.

4. If there exists a strictly dominant equilibrium, it must be the *unique*.

1.2 Domination in mixed strategies and computing equilibrium) - 16 puan

Consider the following two-person game.

	L	R
T	$2, 5$	$0, 0$
M	$0, 0$	$5, 2$
B	$1, 1$	$1, 1$

1. Let p be the probability player 1 plays T and q be the probability that player 2 plays L. What is the range of values p can take so that the mixture of T and M strictly dominates B, i.e. find the set of mixed strategies that strictly dominates B? (4 puan)

2. Given that B is strictly dominated by a mixture of T and M, find and *draw* the best responses of each player and the set of all (pure and mixed) NE. (12 puan)

2 Applications of strategic games - 45 points

2.1 Market games - 30 points

Consider **Cournot** duopoly market game with linear demand $P(Q) = 130 - Q$, where Q is the total quantity, i.e. $Q = q_1 + q_2$. Each firm has constant marginal cost c_i, $C_i(q_i) = 10q_i$ i.e. marginal costs are 10. Each firm *simultaneously* chooses a quantity level to produce.

1. Write down the strategic form of this game. (3 points)

2. Derive and draw the best responses of each firm. Clearly label your graph. Find the Cournot-Nash equilibrium (q^c, q^c) of this game. (15 points)

3. Which choices survive one round of IESDS? What is the rationality requirement for one round of elimination? Justify your answer. (4 points)

4. Which choices survive IESDS? Is this game dominance solvable? What is the rationality requirement (rationality, k-level knowledge, common knowledge)? Justify your answer. (4 points)

5. Is the Cournot-Nash equilibrium Pareto efficient? Justify your answer. (4 points)

2.2 Free-rider (public good) problem - 15 puan

There are two partners in a firm. Each partner chooses independently and simultaneously how much effort to put on the job. The profit of the firm is given by

$$\pi(x, y) = 2(x + y + \frac{1}{2}xy)$$

where x is the amount of effort chosen by partner 1 and y is the amount of effort chosen by partner 2. Assume that x and y have to be between 0 and 4. Partner 1's cost of effort is x^2 and partner 2's cost of effort is y^2. Each partner's payoff is given by his share of profits minus the cost of effort.

1. Find and draw the best response correspondence for each partner. What is the Nash equilibrium? (8 points)

2. Is the Nash equilibrium outcome Pareto efficient? Justify your answer!(3 points)

3. Is there any strictly dominated strategy for any of the players? Is this game dominance solvable? If so, what is the number of iterated elimination? Justify your answer. (4 points)

3 NFG with incomplete information (Bayesian games) - 15 puan

Diyelim ki bir hoca (Ayça) ögrencisine (Ayşe'ye) ödev veriyor. Ayşe ödevi başkasından geçirebilir (plagiarize - P) ya da dürüst davranıp kendisi yapabilir (honest - H). Ayça, Ayşe'nin başkasından çekip çekmedigini kontrol edebilir (check - C) ya da etmeyebilir (not check - N). Ayça, sert (tough) veya yumusak (soft) mizaca sahip olabilir. Ayşe, Ayça'nın mizacını bilmemektedir; ancak $\mu \in (0, 1)$ ihtimalle sert olduguna inanmaktadir. Payoff tabloları şu şekilde verilmiştir (sagdaki tough için):

	C	N
P	0,1	2, 0
H	1,2	1, 3

	C	N
P	0,1	2, 0
H	1,4	1,3

1. *Pure* strategy separating equilibria'yi bulun.

2. *Pure* strategy pooling equilibria'yi bulun.

4 BONUS Drawbacks of NE Solution Concept (10 puan)

1. Strong Nash equilibrium: Consider the following 3-player game where player 1 chooses $\{U, D\}$; player 2 chooses $\{L, R\}$ and player 3 chooses $\{Table1, Table2\}$. Answer the following questions.

	L	R
U	$3, 1, -3$	$-3, -5, 2$
D	$-3, -5, 2$	$3, 1, 9$

	L	R
U	$2, 2, 7$	$-4, -4, 1$
D	$-4, -4, 1$	$-1, -1, 1$

   (a) What are the pure strategy Nash equilibria. **(2 puan)**

   (b) First show that Nash equilibria may not be immune to coalitional deviations. Then, find pure strategy strong Nash equilibria of this game.
   Hint: **Strong Nash** equilibrium strengthens Nash equilibrium by adding the requirement that a strategy profile has to be immune to coalitional deviations. **(3 puan)**

2. Robustness (stability) of Nash equilibrium: Nash equilibrium is not immune to small perturbations in the game. Suppose that players may make mistakes and play each action at least with $\epsilon > 0$ probability. Find the Nash equilibria of the following game. Which one is stable? Justify your answer. **(5 puan)**

	L	R
U	$2, 2$	$0, 1$
D	$1, 0$	$0, 0$