TOBB ETU Department of Economics
ECON 415 Game Theory
Midterm Exam

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Name:.....

- The exam is out of 100 + 10 points.
- The exam ends at 20:15.
- You are not supposed to use a calculator, check your phones, look at your notes, the textbook or others' tests during the exam. Engaging in these activities are all considered as cheating in the exam.
- Please show all your work to get partial credit.
- Allocate your time efficiently.
- Don't forget fully label all graphs.
- Make sure that your exam is 8 pages.

Good luck ⌣̈

1 Normal form games – Basic concepts - 40 points

1.1 Definitions/True-False - 24 points

Consider a normal form game $G =< N, (S_i)_{i \in N}, (u_i)_{i \in N} >$ where $N = \{1, ..., n\}$ is the set of players, S_i is the strategy set for player $i \in N$ and $u_i : S \to \Re$ is the payoff function of player $i \in N$ (attaching a payoff to each strategy profile $s = (s_1, ..., s_n) \in S = \prod_{i \in N} S_i$). Let $s_{-i} \in \prod_{j \neq i} S_j$ denote a strategy profile of all players but player i. State whether the following statements are true or false. Prove if it is true and give a counter example if it is false. Define the terms that you may be using in proofs to get partial credit.

1. Every finite normal form game has a pure strategy Nash equilibrium.
   Her sonlu normal form oyunun pure Nash dengesi vardır.

2. Every finite normal form game has a completely mixed strategy Nash equilibrium.
   Her sonlu normal form oyunda tamamen mixed (pure olmayan) bir Nash dengesi vardır.

3. A strictly dominant action cannot be a completely mixed action.
   Herhangi bir strictly dominant (kesin basan) action profili tamamen mixed (saf olmayan) bir hareket olamaz.

4. If there exists a strictly dominant equilibrium, it must be the *unique*.

1.2 Domination in mixed strategies and computing equilibrium) - 16 puan

Consider the following two-person game.

	L	R
T	$2, 5$	$0, 0$
M	$0, 0$	$5, 2$
B	$1, 1$	$1, 1$

1. Let p be the probability player 1 plays T and q be the probability that player 2 plays L. What is the range of values p can take so that the mixture of T and M strictly dominates B, i.e. find the set of mixed strategies that strictly dominates B? (4 puan)

2. Given that B is strictly dominated by a mixture of T and M, find and *draw* the best responses of each player and the set of all (pure and mixed) NE. (12 puan)

2 Applications of strategic games - 60 points

2.1 Market games - 30 points

Consider **Cournot** duopoly market game with linear demand $P(Q) = 130 - Q$, where Q is the total quantity, i.e. $Q = q_1 + q_2$. Each firm has constant marginal cost c_i, $C_i(q_i) = 10q_i$ i.e. marginal costs are 10. Each firm *simultaneously* chooses a quantity level to produce.

1. Write down the strategic form of this game. (3 points)

2. Derive and draw the best responses of each firm. Clearly label your graph. Find the Cournot-Nash equilibrium (q^c, q^c) of this game. (15 points)

3. Which choices survive one round of IESDS? What is the rationality requirement for one round of elimination? Justify your answer. (4 points)

4. Which choices survive IESDS? Is this game dominance solvable? What is the rationality requirement (rationality, k-level knowledge, common knowledge)? Justify your answer. (4 points)

5. Is the Cournot-Nash equilibrium Pareto efficient? Justify your answer. (4 points)

2.2 First-price and second-price sealed bid auctions - 18 points

Two bidders are involved in a first-price and second-price auction where the valuations for the object is $v_1 > v_2 > 0$. Bidders *simultaneously* submit a bid, which can be any nonnegative number, and the highest bidder wins. In case of a tie, the lowest index individual gets the object. In the first-price auction, if bidder i bids b_i and wins the object, then her payoff is $v_i - b_i$, while is she loses her payoff is 0. In the second-price auction, the winner pays the second highest price, i.e. b_{-i} (since there are only two players), and her payoff is $v_i - b_{-i}$.

1. Show that truthtelling $(b_1, b_2) = (v_1, v_2)$ is a Nash equilibrium in the second-price auction; whereas it is NOT in the first-price. (6 points)

2. Show that first-price auction is efficient (in every NE the player who values the object most gets the object); whereas there may be equilibria when the second-price auction is NOT efficient. (6 points)

3. Although there may be equilibria when the second-price auction is NOT efficient, the *only* weakly undominated Nash equilibrium is efficient since it is the truthtelling equilibrium: $(b_1, b_2) = (v_1, v_2)$. You may show only for player 2. (6 points)

2.3 Free-rider (public good) problem - 12 puan

There are two partners in a firm. Each partner chooses independently and simultaneously how much effort to put on the job. The total profit of the firm which the partners *share equally* is given by

$$\pi(x, y) = 4(x + y + \frac{1}{2}xy)$$

where x is the amount of effort chosen by partner 1 and y is the amount of effort chosen by partner 2. Assume that x and y have to be between 0 and 4. Partner 1's cost of effort is x^2 and partner 2's cost of effort is y^2. Each partner's payoff is given by his share of profits minus the cost of effort.

1. Find and draw the best response correspondence for each partner. What is the Nash equilibrium?

2. Is the Nash equilibrium outcome Pareto efficient? Justify your answer!

3 BONUS Political/Electoral Competition and the Median Voter Theorem (10 puan)

Diyelim ki iki aday (candidate A and B) bir seçimde en çok oyu almak için yarışıyorlar. Adayların aksiyonları (x_A ve x_B) $[0, 1]$ aralığında bir pozisyon seçmek olsun. Her vatandaşın/seçmenin ideal pozisyonu $[0, 1]$ aralığında bulunmaktadır. Seçmenler bu aralıkta uniform dağılmıştır; yani $[0, 1]$ aralığındaki her nokta için ideal pozisyonu bu nokta olan eşit miktarda seçmen bulunmaktadır. Her aday, ideal pozisyonu diğer adayın sunduğu pozisyon ile karşılaştırıldığında kendi sunduğu pozisyona daha yakın olan bütün seçmenlerin oyunu çekmektedir. Bir aday seçimi kazanırsa (oyların majority/çoğunluğunu alırsa) faydası 2, berabere kalırsa (adaylar eşit miktarda oy alırsa) faydası 1 ve kaybederse faydası 0 olarak verilmektedir.

1. Iki aday arasındaki bu seçim oyununu normal-form oyun şeklinde yazın. (players, strategies, payoffs)

2. Her aday için diğer adayın seçtiği pozisyona göre belirlediği en iyi tepki (best response) fonksiyonunu yazın. (Ipucu: Kritik seçmen kimdir?)

3. $(x_A, x_B) = (\frac{1}{2}, \frac{1}{2})$ strateji profilinin (i.e. policy convergence) tek Nash dengesi olduğunu gösterin.(Ipucu: Önce bu profilin Nash olduğunu, daha sonra başka herhangi bir profilin Nash dengesi olamayacağını gösterin.)