# TOBB-ETU Department of Economics 

ECON 415 - FALL 2018
Quiz 1
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## Strategic Form (Normal form) Games with Complete Information

## 1 Definitions/True-False 20 points

Consider a normal form game $G=<N,\left(S_{i}\right)_{i \in N},\left(u_{i}\right)_{i \in N}>$ where $N=\{1, \ldots, n\}$ is the set of players, $S_{i}$ is the strategy set for player $i \in N$ and $u_{i}: S \rightarrow \Re$ is the payoff function of player $i \in N$ (attaching a payoff to each strategy profile $\left.s=\left(s_{1}, \ldots, s_{n}\right) \in S=\prod_{i \in N} S_{i}\right)$. Let $s_{-i} \in \prod_{j \neq i} S_{j}$ denote a strategy profile of all players but player $i$. State whether the following statements are true or false. Prove if it is true and give a counter example if it is false. Define the the bold faced terms!

1. A strategic game that has a weakly dominant strategy equilibrium may have two distinct Nash equilibria. (10 points)
2. A strategic game that has a strictly dominant strategy equilibrium may have two distinct Nash equilibria. (10 points)

## 2 Domination and finding NE with mixed strategies - 30 points

Chicken game or Dove-Hawk game: Telliog̃ulları ve Seferog̃ulları aileleri Yeşil Vadi üzerinde anlaşmazlıg̃a düşüyorlar. Vadinin deg̃eri $v>0$ olsun. Kavga etmenin maliyeti (kol, bacak kırılması vb. gibi hasarlar görüldügü için) ise $c>0$ ile gösterilsin. Eg̃er iki taraf agresif/sahince (hawkish) davranırsa kavga ediyorlar (kavganın maliyetini ödemek zorunda kalıyorlar) ve vadiyi paylasıyorlar ( $v / 2$ ). Iki taraf da barışçıl (dovish) yollarla cözüm bulurlarsa vadiyi paylaşıyorlar ve kavga maliyeti ödemek zorunda kalmıyorlar. Eg̃er bir taraf agresif, dig̃er taraf barışçıl davranırsa, agresif olan bütün vadiyi alıyor (kavga maliyeti ödemiyor).

1. Bu oyununun ödül matriksini (payoff matrix) yazın. Oyuncunun aksiyonları: $H$ (hawkish) ve $D$ (dovish). (7 puan)
2. Bu oyunda hawkish aksiyonunun kesin baskın strateji olmaması için gerek şart nedir? (5 puan)
3. Diyelim ki $v=4$ ve $c=3$. Bu durumda Nash dengeleri (pure ve mixed) nelerdir? Best response'ları çizerek bulun.(18 puan)

## 3 Economic Application - 50 points

Political campaigning - $\mathbf{2 0}$ points Two candidates are competing in a political race. Each candidate $i$ can spend $s_{i} \geq 0$ on ads, which increases the probability that candidate $i$ wins the race. Given a pair of spending choices $\left(s_{1}, s_{2}\right)$, the probability that candidate $i$ wins is given by $\frac{s_{i}}{s_{1}+s_{2}}$. If neither spends any resources then each wins with probability $\frac{1}{2}$. Each candidate values winning at a payoff of $v>0$ and the cost of spending $s_{i}$ is just $s_{i}$.

1. Write this situation as a normal form game. (Hint: Spending choices are strategies. What is the expected payoff of candidate $i$ given the strategy profile $\left(s_{i}, s_{j}\right)$ ?) (6 points)
2. Find the unique symmetric Nash equilibrium $\left(s_{1}^{*}, s_{2}^{*}\right)$. What is the payoff players attain at the Nash equilibrium profile? What happens to the Nash equilibrium spending (for ads) levels as $v$ increases? (10 points)
3. Is the Nash equilibrium Pareto efficient? Justify your answer. (4 points)

First-price and second-price sealed bid auctions - $\mathbf{3 0}$ points Two bidders are involved in a first-price and second-price auction where the valuations for the object is $v_{1}>v_{2}>0$. Bidders simultaneously submit a bid, which can be any nonnegative number, and the highest bidder wins. In case of a tie, the lowest index individual gets the object. In the first-price auction, if bidder $i$ bids $b_{i}$ and wins the object, then her payoff is $v_{i}-b_{i}$, while is she loses her payoff is 0 . In the second-price auction, the winner pays the second highest price, i.e. $b_{-i}$ (since there are only two players), and her payoff is $v_{i}-b_{-i}$.

1. Show that truthtelling $\left(b_{1}, b_{2}\right)=\left(v_{1}, v_{2}\right)$ is a Nash equilibrium in the second-price auction; whereas it is NOT in the first-price. (15 points)
2. Show that first-price auction is efficient (in every NE the player who values the object most gets the object); whereas there may be equilibria when the second-price auction is NOT efficient. (15 points)

## 4 BONUS: More TRUE/FALSE - 10 points

1. State whether a Nash equilibrium that involves play of a weakly dominated action must be perfect i.e. it is robust to small perturbations in the game or not (8 points). Prove if your answer is YES, construct an example if it is NO.
2. State whether there is a strategic game that has a strictly dominant strategy equilibrium that is NOT a Nash equilibrium. Justify your answer i.e. construct one if your answer is YES, prove there cannot be one if your answer is NO.
