TOBB-ETU Department of Economics

ECON 415 - SUMMER 2017
Quiz 1

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Strategic Form (Normal form) Games with Complete Information

1 Definitions/True-False 24 points

Consider a normal form game $G =< N, (S_i)_{i \in N}, (u_i)_{i \in N} >$ where $N = \{1, ..., n\}$ is the set of players, S_i is the strategy set for player $i \in N$ and $u_i : S \rightarrow \Re$ is the payoff function of player $i \in N$ (attaching a payoff to each strategy profile $s = (s_1, ..., s_n) \in S = \prod_{i \in N} S_i$). Let $s_{-i} \in \prod_{j \neq i} S_j$ denote a strategy profile of all players but player i. State whether the following statements are true or false. Prove if it is true and give a counter example if it is false. Define the the **bold faced** terms to get partial credit.

1. A **strictly dominated action** cannot be a part of Nash equilibrium. (8 points)

2. A **weakly dominated action** cannot be a part of Nash equilibrium. (8 points)

3. One cannot construct a 2×2 NFG having two action profiles that give a payoff of 5 to both players for which one strategy profile is a **Nash equilibrium** while the other is not. (8 points)

2 Drawbacks of NE solution concept - 16 puan

1. Strong Nash equilibrium: Consider the following 3-player game where player 1 chooses $\{U, D\}$; player 2 chooses $\{L, R\}$ and player 3 chooses $\{Table1, Table2\}$. Answer the following questions.

	L	R
U	$3, 1, -3$	$-3, -5, 2$
D	$-3, -5, 2$	$3, 1, 9$

	L	R
U	$2, 2, 7$	$-4, -4, 1$
D	$-4, -4, 1$	$-1, -1, 1$

   (a) What are the pure strategy Nash equilibria. **(4 points)**

   (b) First show that Nash equilibria may not be immune to coalitional deviations. Then, find pure strategy strong Nash equilibria of this game. Hint: **Strong Nash** equilibrium strengthens Nash equilibrium by adding the requirement that a strategy profile has to be immune to coalitional deviations. **(4 points)**

2. Robustness (stability) of Nash equilibrium: Nash equilibrium is not immune to small perturbations in the game. Suppose that players may make mistakes and play each action at least with $\epsilon > 0$ probability. Find the Nash equilibria of the following game. Which one is stable? Justify your answer. **(8 points)**

	L	R
U	$2, 2$	$0, 1$
D	$1, 0$	$0, 0$

3 Domination and finding NE with mixed strategies - 20 points

Tellioğulları ve Seferoğulları aileleri Yeşil Vadi üzerinde anlaşmazlığa düşüyorlar. Vadinin değeri $v > 0$ olsun. Kavga etmenin maliyeti (kol, bacak kırılması vb. gibi hasarlar görüldügü için) ise $c > 0$ ile gösterilsin. Eğer iki taraf agresif/sahince (hawkish) davranırsa kavga ediyorlar (kavganın maliyetini ödemek zorunda kalıyorlar) ve vadiyi paylasıyorlar ($v/2$). Iki taraf da barışçıl (dovish) yollarla cözüm bulurlarsa vadiyi paylaşıyorlar ve kavga maliyeti ödemek zorunda kalmıyorlar. Eğer bir taraf agresif, diğer taraf barışçıl davranırsa, agresif olan bütün vadiyi alıyor (kavga maliyeti ödemiyor).

1. Bu oyununun ödül matriksini (payoff matrix) yazın. Oyuncunun aksiyonları: H (hawkish) ve D (dovish). (4 puan)

2. Bu oyunda *hawkish* aksiyonunun kesin baskın strateji olmaması için gerek şart nedir? (3 puan)

3. Diyelim ki $v = 4$ ve $c = 3$. Bu durumda Nash dengeleri (pure ve mixed) nelerdir? **Best response'ları çizerek** bulun.(13 puan)

4 Economic Application - 40 points

Tragedy of commons – public good problem: Suppose that there are two firms each choosing how much to produce *simultaneously*. Each production consumes some of the clean air. There is a total amount of clean air that is equal to K and the consumption of clean air comes out of this common resource. Each player i (firm) chooses its own consumption of clean air for production, which is denoted by $k_i \geq 0$. The amount of clean air left is $K - \sum_{j=1}^{2} k_j$. The firm enjoys not only the consumption of the clean air for its production but also the clean air left after the production. Thus, its payoff function is given as:

$$u_i(k_i, k_{-i}) = \ln(k_i) + \ln(K - \sum_{j=1}^{2} k_j)$$

Answer the questions below for this environment.

1. Describe this situation as a strategic game.

2. Compute and draw the best response correspondence for each firm. Then find the NE.

3. Is the Nash equilibrium outcome Pareto efficient? If not, give an example of an efficient strategy profile.

4. Which actions survive one round of iterated elimination of strictly dominated actions? What is the rationality requirement for one round of iteration? Justify your answer.

5. Which strategy profiles survive IESDS? Is this game dominance solvable? What is the rationality requirement (rationality, k-level knowledge, common knowledge)? Justify your answer.

5 BONUS: - 10 points

(War of Attrition) Consider a situation where there are two parties disputing over an object. Assume that the value party i attaches to the object is $v_i = v > 0$. Let time be a continuous variable that starts from 0 and runs indefinitely. Each unit of time that passes before the dispute is settled (i.e. one of the parties concedes) costs each party one unit of payoff. Thus, if player i concedes first, at time t_i, her payoff is $-t_i$ (she spends t_i units of time and doesn't obtain the object.) If the other player concedes first, at time t_j, player i's payoff is $v_i - t_j$ (she obtains the object after t_j units of time). If both players concede at the same time, player i's payoff is $\frac{1}{2}v_i - t_i$ where t_i is the common concession time.

1. Write down the strategic form of this game.

2. Derive and draw the best response correspondences and find the set of Nash equilibria.